Chapter 2

A Model for Mathematics Instruction to Enhance Student Motivation and Engagement

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> *I like the way my Year 8 mathematics teacher doesn't just mark my work right or wrong—she actually explains what I need to do to improve. She gives me confidence so I know I can do this!*
>
> —Thirteen-year-old female student

An underlying message of the National Council of Teachers of Mathematics (NCTM) *Principles and Standards for School Mathematics* (2000) is the necessity to build students' confidence and motivation in mathematics. Aligned with other international mathematics reform literature (e.g., Australian Education Council 1990), *Principles and Standards* advocates teaching practices presumed to enhance motivation and engagement because they not only are considered desirable outcomes by themselves but also are a means to enhance student achievement. A substantial body of research now confirms that many teaching strategies designed to increase student motivation at school, particularly in mathematics, also improve learning outcomes (Stipek et al. 1998). These findings are especially relevant for teachers of students in the middle grades. The middle grades coincide with a notable decline in students' engagement levels with school and with mathematics in particular (Martin 2007, 2008). It is also a time when progression of learning is prone to lose its momentum compared with that in elementary school (Hill, Holmes-Smith, and Rowe 1993). Therefore, giving teachers guiding frameworks—to help them (and their students) develop a deeper understanding of motivation and the relationships between certain teaching practices and student achievement, motivation, and engagement in mathematics—is essential. This chapter focuses on a multidimensional framework to articulate instructional strategies that will enhance middle-grades students' engagement and motivation in mathematics.

What Is Motivation?

Holmes (1990) described motivation as the "fuel" for mathematics learning, viewing it as an "essential component of mathematics instruction" (p. 101) along with effective teachers and quality teaching. Motivated students have the energy and drive to learn and the thoughts and behaviors that reflect this energy and drive (Martin 2007, 2010). Accordingly, if educators wish to address all aspects of student motivation, they must focus on both the mind—student beliefs and expectations—and their actions, such as students' abilities to plan, manage, and persist at tasks such as homework and challenging problems.

Requests from teachers or parents to study harder or to engage more with class activities can be meaningless if students do not understand what motivation and engagement really mean in the school context and lack explicit strategies for achieving them. Furthermore, a common understanding among parents, educators (making such requests of their children), and the students themselves of what these concepts mean, and of specific strategies and advice to improve student motivation, would be helpful.

Some questions arise: How can knowledge of a theoretical framework for motivation and engagement help address our concerns about student motivation in mathematics? What teaching strategies might motivate middle-grades students to engage more in mathematics? Before looking at specific teaching strategies, exploring important aspects of motivation and engagement as part of a unified framework is useful.

The Motivation and Engagement Wheel

Motivation's sheer diversity and fragmentation is one of its limitations (Murphy and Alexander 2000; Pintrich 2003). Educational research does not always yield useful applications, and a need exists to combine advances in scientific understanding with applied utility. Accordingly, previous work has recommended giving more attention to "use-inspired basic research" (Stokes 1997; Pintrich 2003). With this in mind, Martin (2007, 2008) developed the Motivation and Engagement Wheel to capture an integrative framework that represents seminal motivation and engagement theory. The Motivation and Engagement Wheel emerged through an attempt to bridge the gap between diverse educational theorizing and practitioners' (e.g., teachers, counselors, psychologists) needs to operate within a parsimonious educational framework that they can also clearly communicate to students. Two conceptual bases underlie the wheel: (1) the general, comprising adaptive cognition, adaptive behavior, impeding/maladaptive cognition, and maladaptive behavior, and (2) the specific, comprising eleven factors subsumed under the general. Figure 2.1 shows the wheel. Above the horizontal axis are the adaptive cognitive and behavioral factors. Below the axis are the impeding/maladaptive cognitive and behavioral factors. Thus, clockwise from left to right are adaptive cognition, adaptive behavior, impeding/maladaptive cognition, and maladaptive behavior.

Although the wheel's general dimensions (adaptive cognition, adaptive behavior, impeding/maladaptive cognition, maladaptive behavior) have been important for research and theory, its specific dimensions—that is, the eleven factors of multidimensional motivation and engagement—are particularly relevant to practitioners. Pintrich (2003) proposed seven questions to guide the development of motivational science. These questions underscored the importance of articulating a model of motivation from psychoeducational theory related to self-efficacy, attributions, valuing, control, self-determination, goal orientation, need achievement, self-regulation, and self-worth. According to Martin (2007), these concepts offered a useful means for identifying the eleven specific factors for the wheel.

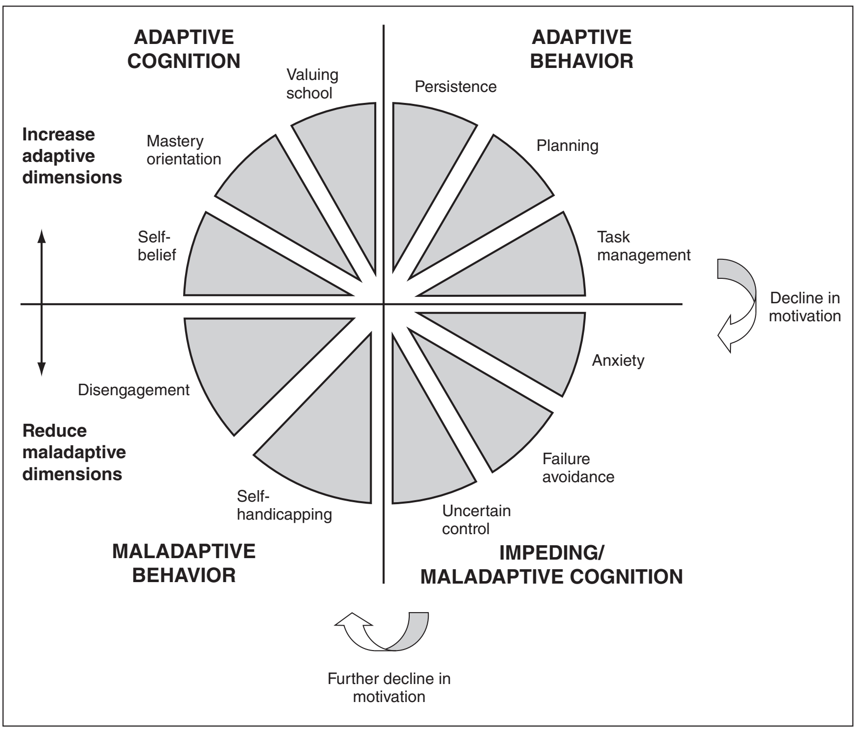

Fig. 2.1. Motivation and Engagement Wheel (Martin 2010, p. 9; reproduced with permission)

As Martin (2007) discusses, (*a*) the wheel's self-efficacy dimension reflects self-efficacy theory (e.g., Bandura 1997); (*b*) the valuing school dimension reflects valuing (e.g., Wigfield and Eccles 2000); (*c*) the mastery orientation dimension reflects self-determination (in the context of intrinsic motivation) and motivation orientation (e.g., Dweck 1986); (*d*) the planning, study management, and persistence dimensions reflect self-regulation (e.g., Zimmerman 2002); (*e*) the uncertain-control dimension reflects attributions and control (e.g., Weiner 1984); and (*f*) the failure avoidance, anxiety, self-handicapping, and disengagement dimensions reflect the need achievement and self-worth theories (e.g., Covington 1992).

Conceptualizing the wheel into the adaptive, impeding, and maladaptive dimensions allows educators to identify a set of common factors that underpin particular students' motivation and engagement, as well as which aspects teachers need to target for intervention. The practitioner and student can easily separate the helpful (adaptive) motivation from the unhelpful (impeding and maladaptive). Thus, this model, although representing a complex aggregation of theory, is an easy way for students to understand their motivation and a convenient way for practitioners to explain it to them. When students understand motivation and the dimensions that make it up, intervention is more meaningful to them and therefore is likely to be more successful.

The adaptive dimensions of the framework reflect enhanced motivation. Hence, we need students to develop self-efficacy in their abilities to understand the mathematics taught and believe that more effort will positively affect their learning. Students who break down difficult or seemingly insurmountable tasks into more achievable chunks realize that they can control and manage their learning more successfully. Thus, they are more likely to persist when they face challenging problems.

Factors in the maladaptive and impeding dimensions reflect reduced motivation and engagement. For example, feelings of anxiety can emerge when students do not understand content, are unsure how to start a task, or consider a task too daunting to even begin. Alternatively, they may use self-handicapping strategies such as doing things other than homework first or leaving the challenging academic tasks until they are too tired to do them effectively. Continued use of such behaviors can cause students to disengage from mathematics. Students will express feelings of giving up and helplessness, possibly withdrawing from the subject and even school in general unless something breaks the downward spiral.

The wheel is only the first step in addressing student motivation. The second step is to assess students on each dimension of the wheel, which is why Martin (2008) developed the Motivation and Engagement Scale (MES). The MES comprises forty-four items (four items for each specific factor in the wheel) and has proven to be a reliable and valid means of assessing students' motivation and engagement—including the role of motivation and engagement in important educational processes and outcomes such as academic (including mathematics) achievement, educational attainment, and pedagogy (Martin 2007, 2010). Together, the wheel and the MES offer a basis for educational practice and intervention aimed at more structured approaches to enhancing student motivation. Following are some guidelines and practical teaching strategies that derive from this framework.

Promoting Motivation in Middle-Grades Mathematics Classrooms

A National Research Council–Mathematics Learning Study Committee report proposed that mathematical proficiency consists of five integrated competencies: conceptual understanding, procedural fluency, strategic competency, adaptive reasoning, and productive disposition (Kilpatrick, Swafford, and Findell 2001). The report defines productive disposition as "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy" (p. 116). The notion of productive disposition aligns well with the adaptive cognition and adaptive behavior quadrants of the Motivation and Engagement Wheel, identifying such factors as valuing school, self-efficacy, persistence, and task management as crucial contributors to positive inclinations toward engaging with learning. How can teachers promote these constructive factors in the middle-grades mathematics classroom? How can we guard against maladaptive behaviors and impeding cognition, such as uncertain control and anxiety? The following section answers these questions by presenting four examples of approaches to teaching and highlighting ways to select the mathematical content, nurture positive classroom relationships, and encourage productive learning behaviors.

Variety in Teaching Approaches

The argument for using variety in teaching approaches to promote motivation and engagement begins with accepting students' individuality. Teachers need to allow for a range of personalities, learning preferences, modes of expression, and work rates. The argument continues with the notion that consistently using the same teaching approach advantages some students and disadvantages others according to the compatibility of the approach with their learning preferences. Injecting variety into teaching methods, resources used in mathematics lessons, and assessment strategies can accommodate a range of student preferences.

One can achieve variety in teaching methods by using various combinations of teacher-centered and student-centered approaches. Teacher-centered strategies include worked examples, explication, demonstration, and structured questioning. Student-centered strategies include collaborative group work, practical tasks, problem solving, open tasks, investigation, games, and student presentations.

Variety in resources involves selecting different modes of task and information delivery, as well as different materials and tools to model and explore mathematical concepts and processes. Such resources include multimedia, computer software, concrete manipulatives, models, simulations and experiment apparatus, calculators and data loggers, newspapers, excursions, and outdoor activities.

Variety in assessment strategies means using a range of formal and informal opportunities for students to demonstrate understanding before, during, and after learning events. These strategies include verbal explanation, visual presentations, concrete models, written records, tests and quizzes, work sample portfolios, self-assessment, and mastery checklists. Because assessment gives not only information for the teacher but also feedback to the student, it can affect both intrinsic and extrinsic motivation (Wiliam 2007).

Table 2.1 presents examples of three teaching methods, three resources, and three assessment strategies for teaching about multiplying decimals by powers and multiples of 10. Although we have aligned the teaching methods, resources, and assessment strategies to create three distinct teaching approaches, teachers could reconfigure the components in many productive combinations. Adjusting teaching and assessment approaches to add variety and accommodate various learning styles in these ways can develop students' self-beliefs in their capacity for success, particularly with individualized tasks and formative feedback, giving a clear direction for further learning.

Real and Relevant Tasks

Opportunities exist across the mathematics curriculum for the teacher to choose a context for tasks. The data/statistics strand of the curriculum offers an excellent example for content selection that is both relevant to the students and based in a real-life context. Content selection begins with getting to know the students. The teacher must find out about students' current interests and concerns inside and outside school, such as music, sports, movies, and local and world events. Every classroom has students who have developed considerable expertise in topics that capture their interest. Using existing interests and context knowledge not only can support engaging those particular students in the mathematical aspects of the statistical task but also creates the opportunity to hold other students' attention as they discover nonmathematical information while developing mathematical ideas and skills. Therefore, interaction between the students, within thoughtfully composed cooperative groups, becomes an important implementation factor. For example, in their Australia–United States investigation of the role of context in

Table 2.1
Example teaching approaches for multiplying and dividing decimals (whole numbers and fractions) by 10, 100, 1000, and other multiples of 10

Approach	Teaching method	Resources	Assessment
1	Student research and collaborative knowledge building: what is the procedure and how/why does it work?	Internet (tutorials, games, YouTube, Google video); class wiki or blog for gradually building a class product to answer the question	Student self-assessment of learning and production of worked examples by individuals or pairs of students
2	Problem solving: group investigation of patterns in number sequences to discover the rule and creation of new patterns; teacher furnishes scaffolding as needed	Calculators	Teacher observation and group oral reports to the class
3	Teacher explanation of the procedure emphasizing place value, followed by written student exercises	Base-10 material and worksheets	Individual work samples

students' analysis of data, Nisbet, Langrall, and Mooney (2007) selected data sets (from the Internet) on two topics of interest to a group of middle-grades students and for one topic thought to be of little interest to them. The researchers devised tasks in the form of problems, with a view to stimulating higher-order thinking, and assigned students to small groups (fig. 2.2). Knowledge of context was an important factor contributing to students' engagement in statistical tasks, with students using context knowledge "to rationalize the data or their interpretations, in taking a critical stance toward the data" (Nisbet, Langrall, and Mooney 2007, p. 16). In the third task (based on Olympic discus-throwing distances), the topic lacked a class expert and drew minimal interest, so much less supportive discussion of relevant context factors took place. Embedding mathematical learning in real and relevant contexts, particularly contexts that let students use personal expertise, can increase their sense of control and self-worth.

Open-Ended Tasks

As well as learning to use mathematics in a range of contexts, students need to

Task 1: Does the USA produce better men tennis players than Australia?

Data sets: (*a*) a table listing information on the winners and runners-up for Wimbledon 1955–2004 and (*b*) a table of the top twenty male tennis players with rankings and prize money

Task 2: In a recent pop-culture survey, teenagers identified Britney Spears and Delta Goodrem as two of the top female performers. From the following data collected from the top 40 charts, which of these two singers is more popular?

Data sets: (*a*) a table for each singer of top 40 hits 2002–2005, including rankings and number of weeks, and (*b*) tables of top 40 ranking from charts around the world for the two singers

Fig. 2.2. Tasks to stimulate higher-order thinking (adapted from Nisbet, Langrall, and Mooney 2007)

develop deeper understanding for a range of important mathematical concepts (Kilpatrick, Swafford, and Findell 2001). Students also need to develop procedural fluency. To this end, teachers often use repetitive exercises to promote practice; overusing this approach may lead to student disengagement.

Another strategy that creates increased opportunities for motivation and engagement involves actively exploring mathematically rich situations through open-ended questions and tasks. Open-ended questions are more readily accessible since all students can start from their current knowledge and understanding. When planning to use open-ended questions, teachers need to design enabling prompts to support students who falter after only one or two solutions as well as additional prompts to extend thinking for students who complete the task more quickly (Sullivan, Mousley, and Zevenbergen 2006). Sharing problem-solving strategies in small-group situations can also enhance students' extended thinking.

Teachers can easily create open-ended questions by working backward from an answer or adapting a standard question (Small 2009; Sullivan and Lilburn 2004). The first approach involves selecting a topic and considering an answer to a typical question. Working backward from this answer, one can then derive open-ended questions. For example, when ordering fractions and giving students pairs of fractions to compare (e.g., "Which is bigger, $^5/_{12}$ or $^3/_5$?") a teacher may ask students to list some fractions less than $^3/_5$. By posing the additional challenge "Take one of your answers and explain how you know that it is less than $^3/_5$," teachers gain valuable assessment information about students' thinking and choice of strategies. If students find determining solutions difficult, some enabling prompts could include using a number line to locate $^3/_5$ or replacing $^3/_5$ with a simpler fraction such as $^1/_2$. A prompt to extend thinking would be to have students find fractions between $^2/_5$ and $^3/_5$.

The second approach involves adapting a standard question. For example, if students are learning about the properties of quadrilaterals, instead of using the question "What is a parallelogram?" the teacher could adapt the question by asking students to write down everything that they know about a parallelogram. Students having difficulty could begin by recording what they know about a rectangle and then comparing a rectangle with a parallelogram. The teacher can further engage students who easily complete the task by posing the additional challenge, "Compare and contrast the properties of a parallelogram with those of other quadrilaterals."

Open-ended questions may have many solutions and several methods of finding solutions. Such a format opens up opportunities for success, reducing anxiety and giving students more control as they decide what strategies to use. Open-ended questions also promote developing problem-solving skills and are generally considered less threatening than more challenging problem tasks. Instead of the "correct answer," the focus shifts to process, independence, flexibility, and persistence.

Using Errors as a Focus for Learning in Mathematics Lessons

Acknowledging the role of errors and misconceptions in the learning process also lets students take control of their learning and self-assess. Many factors may cause students' errors in mathematics. Poor comprehension, language difficulties, anxiety, rushing, and carelessness may lead to errors in completing tasks. However, misconceptions from overgeneralizing rules or failure to connect new ideas to existing knowledge and understanding usually cause systematic errors. Just as teachers review student scripts to look for patterns in errors, students need encouragement to review their errors, reinforcing the belief that making errors is an important part of learning. Changing beliefs such as "you cannot learn from your mistakes" will be necessary if students are to overcome failure avoidance, anxiety, and self-handicapping in mathematics.

Students' early mathematics learning may lead them to make generalizations such as "multiplication makes bigger," "division involves dividing a bigger number by a smaller number," and "longer numbers are always greater in value." Although these generalizations may be satisfactory with whole numbers in the early elementary grades, they may not apply for fractions and decimals. Teachers need to explicitly discuss such statements so that students are more aware of common misconceptions. Teachers could present one of these statements to students at the beginning of a lesson to encourage them to seek counterexamples.

A related teaching strategy is to pose a statement and ask students whether it is always, sometimes, or never true (Bills et al. 2004). Students discuss the possibilities in small groups, identifying examples and counterexamples to support their arguments. Further examples of statements that present opportunities for rich discussion include the following:

- Parallelograms have no axes of symmetry.
- To multiply a number by 10, you put a zero on the end.
- The larger the area of a shape, the larger the perimeter.
- All four-sided shapes tessellate.

Conclusion

Many teaching strategies that increase student motivation also positively influence academic achievement. Because both achievement and motivation are prone to falter in the crucial middle grades, educators must increase their understanding of these dimensions and their relationship to effective teaching strategies. In this chapter, we presented a use-inspired multidimensional framework for understanding motivation with direct applicability to teachers, parents, and students. The Motivation and Engagement Wheel can offer direction for mathematics instruction at the whole-class level and for developing targeted interventions for enhancing individual students' motivation. We have also suggested possible directions for what whole-class instructional approaches and targeted intervention strategies might look like through four examples: variety in teaching approaches, real and relevant tasks, open-ended questions, and using errors as a focus for learning. Building students' motivation in mathematics to enhance mathematics achievement, and as a vital end in itself, is an essential component of mathematics instruction.

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