

Understanding the Empty Number Line

Associate Professor Janette Bobis

Mathematics Education, Faculty of Education and Social Work

Associate Dean, Division of Research

University of Sydney

It really is essential that children have a solid understanding of mathematics while in primary school and often the support provided at home can make an enormous difference to their overall performance.

My name is Janette Bobis and I teach mathematics education to prospective primary teachers in the Faculty of Education and Social Work at the University of Sydney. I want to talk about a new tool or model that is being used in primary classrooms to assist students develop more efficient mental computation strategies. This tool is called the 'empty number line', and sometimes the 'blank number line'. Knowing more about the strategies teachers use to support learning in the classroom helps parents feel more confident when discussing their child's progress with their teacher or when providing some assistance with homework.

Mental computation is now receiving greater emphasis in the curriculum and in classrooms across the world than ever before. We've known for quite some time about the importance of being able to calculate mentally and recognised that 'good' mental calculators in the past were quite flexible in their strategy use and generally used their own 'invented' mental strategies — those that were not taught at school or conformed to any traditional standard pencil and paper method. Such 'invented' strategies are potentially quite powerful because they make sense to the user. But not all invented strategies are 'efficient' in all situations. For example, to determine the answer to $34 + 3$, a child might start at 34, and then count by ones ... 35, 36, 37. Counting-on by ones might be efficient in this case, but using the same strategy to calculate $34 + 26$ is not.

In classrooms today, teachers will guide students who would not ordinarily develop more sophisticated strategies by themselves to progress to using more efficient ones. More efficient strategies are normally those that don't rely on 'just' counting-by-ones. This is where the empty number line can be helpful as it supports the development of non-count-by-one strategies.

Let's start by looking at the structured number line, as this is one model that most of us are more familiar with.

The structured number line is a 'measurement model'. By this, we mean that the numbers are representations of lengths, rather than simply points on a line that they are labelling. The lengths or proportion of the line between the numbers and marks provide clues as to the value of a missing number.

On the other hand, the empty number line is a 'counting model'. The marks on the empty number line are not meant to be proportional. This means that the lengths or the distances between the marks will not indicate their value. That's because the empty number line's purpose is very different from that of the structured number line.

Starting with an empty line — a number line with no numbers or markers — students only mark the numbers they need for their calculation. For example, if using an empty number line to assist solving $58 + 24$, a student would start with 58, jump ahead 20 to 78, then jump ahead 4 more using either ones or a jump of 2, then 2 more to reach 82. You'll note that zero, or any other unnecessary number, is not represented on the empty number line.

To understand why the empty number line was introduced into our curriculum it's helpful to know a little about its origins. The empty number line was originally proposed as a model for addition and subtraction in the Netherlands in the 1980s and it's now regularly used in classroom instruction throughout the Netherlands. Since then, the use of empty number lines has been advocated by curriculum documents around the world, particularly in the UK, New Zealand and in states and territories across Australia.

It was originally developed out of a need for a 'new' tool to help overcome problems experienced by children when performing addition and subtraction involving two-digit numbers. Often, when children are introduced to a procedure for solving a problem — when they may not have a deep understanding of the mathematics underlying the process — a common result is a robot-like implementation of a procedure that makes no sense to the child.

For example, a common error made when solving $53 - 26$ using either blocks to model the process or a traditional pencil and paper procedure, is for a child to calculate the digits in the ones column first. Having learnt from an early age that they must take the 'little' number away from the 'larger' one, children commonly make the mistake of reversing the order and take 3 from 6.

When the empty number line is used to support a solution strategy, this error is unlikely to be made, since the calculation proceeds in a linear fashion. Starting at 53 on the empty number line, the child would jump back 20 (or two lots of 10) to 33, then jump back another 6, possibly using two lots of 3 jumps to arrive at 27.

In this way, the empty number line reinforces the use of a very powerful mental strategy for calculations involving two digits or more. The 'jump strategy' that is supported by the empty number line model treats the first number as a 'whole' — it's not split into tens and ones as required when traditional pencil and paper procedures are used. The second number can then be partitioned in any number of convenient ways to assist addition or subtraction. For instance, in the case of $48 + 25$, a student might choose to jump from 48 to 58, then 68 using two lots of 10s, then add 5 more using five jumps of one. A student with more experience using the strategy, and a more advanced understanding of number relationships, might start at 48, make one jump of 20 to 68, then partition the 5 into a 2 and a 3.

In this way, the student could conveniently build the 68 to the next 10 — being 70 — and finally add the 3 to make 73. There are other sequences of jumps that could be used also to derive the correct answer, but the important thing is that students choose combinations that make sense to them.

The empty number line is a model that not only scaffolds students' thinking to more abstract levels, but also allows that thinking to become visible to others and to themselves. The line can scaffold children's thinking because it allows them to see what parts have been calculated and what parts remain, thus reducing the load on a child's working memory.

The line encourages children to develop more efficient mental strategies for working with numbers that are two digits or larger. It reinforces a jumping strategy that involves counting in multiples of tens and other non-count-by-one approaches.

Eventually though, the student must be able to mentally compute without the assistance of the empty number line or any other visible support mechanism.

An important prerequisite for children's success in mental computation is a thorough understanding of numbers to 100. This includes counting skills — first by ones and then counting by multiples, especially multiples of 10.

It's important that we encourage children to count both forwards and backwards and that they can start counting from any number, not just from one. My own children used to count-back with the clock on the microwave oven while waiting for food to cook. They can also practise counting-on if supervised to program a microwave's cooking time by adding seconds and minutes in multiples of tens and ones on the keypad.

At first, a structured number line can be used to support children as they learn to count and sequence numbers. Any opportunity to reinforce the correct sequence of numbers, either when counting by ones, tens or any other multiple, will be a benefit. The calculator is particularly powerful in helping

children learn to count this way. For example, a child could key in the numeral '6' then 'plus 10', then keep pressing the 'equals' button and the calculator will count 6, 16, 26, 36 ... and so on, counting in tens, but off the decade. Counting in this way is an essential skill for mental computation involving larger numbers.

Always essential, is knowing the basic facts. To enable children to flexibly partition numbers when dealing with larger numbers, they must first be able to recognise that a '5' for instance, can be made in multiple ways: $3 + 2$; $4 + 1$; and $7 - 2$; and many others. If children understand these number relationships, they will be able to 'see' that to solve $8 + 5$, they could partition the 5 into $2 + 3$, and add it to the 8 in parts. The 2 can be added to the 8 to make 10 before the final part is added. So, $8 + 2 = 10$; and then $10 + 3 = 13$.

Parents can help by encouraging their children to verbalise their strategies. Ask them to explain how they calculated answers or to explain how they used the empty number line to solve certain problems.

Remember, the ultimate goal is for students to effectively calculate mentally without the need for external support models. So if your child can already efficiently calculate in their head without using an empty number line, then they probably no longer need the visual support and we would be delaying their progress if we insisted that they use it.

For adults who already 'know' the mathematics, the empty number line can seem like a straightforward, convenient model to assist children with mental calculation. But for children who don't yet have the same level of conceptual understanding, it's still a challenge to learn to use it correctly. And for some children, this will take time and a great deal of support.